IBM TORINO · **133 QUBITS** · APRIL–MAY 2025

The textbook said HHL can’t run on today’s quantum hardware.

It can — if you stop letting the transpiler touch the whole chip, and you never measure a qubit the unitary never used.

A hardware constraint, and the fix.

SWIPE →

The accepted limit

On NISQ devices, the **Quantum Phase Estimation** stage of HHL would execute. The full linear-solver — even on a toy problem — would not.

> The test problem: invert a spectral operator **H_E** built from the elliptic curve **11a1** — the smallest-conductor curve, tied to the **Birch–Swinnerton-Dyer** conjecture. Solve $H_E|x\rangle = |b\rangle$ on real hardware.

Every full-HHL job came back the same way ↓

```
IndexError: index out of range
```

QPE: fine. HHL: dead on arrival.

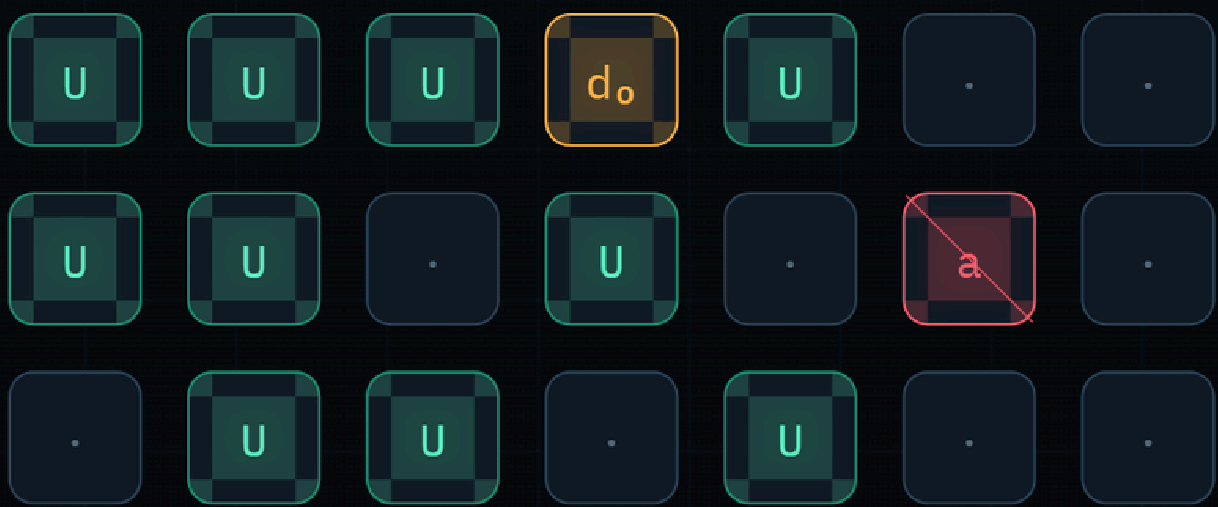
What broke, and what didn't

CIRCUIT UNDER TEST	RESULT
QPE + data-qubit readout	PASS
Minimal CRY gate (2 qubits)	PASS
HHL, clock-only measurement	FAIL
QPE + CRY on ancilla	FAIL
QPE + ancilla read, *no gates on it*	FAIL

> The last row is the tell. Same working QPE circuit. Add **one ancilla qubit**, apply **nothing** to it, just measure it — and the whole job fails.

The gate wasn't the problem. The lone qubit was.

Measuring a qubit the unitary never touched takes the run down.



- in the controlled-U structure
- d_0 — kept, and measured
- a — orphan ancilla → IndexError

Likely a transpiler routing / classical-register mapping edge case for qubits outside the main circuit structure. From the user's side, the rule is simpler:

> **Every measured qubit must participate in at least one controlled operation.**

Delete the orphan

Standard HHL hangs the eigenvalue rotation on a **separate ancilla**. V5 hangs it on **d_0** — a data qubit that's already inside every controlled-U. Nothing orphaned, nothing to break.

```
standard   CRY(θ): |c⟩ ⊗ |a⟩  →  Ry(θ)|a⟩
V5         CRY(θ): |c⟩ ⊗ |d₀⟩ → Ry(θ)|d₀⟩
```

- **+** Constrain the circuit to only the qubits the problem needs.
- **+** Restrict the transpiler from routing across the full 133-qubit chip.
- **+** Measure only qubits in the unitary — drop the orphans.

Trade-off: solution amplitude mixes with the rotation signal. Acceptable on NISQ.

It ran. Cleanly.

5
logical qubits · 3 data + 2 clock

210
transpiled gate depth

1024
shots

28
unique measurement outcomes

Clock register, top outcomes — phases 1·2·3 dominate, matching the non-zero eigenvalues of H_E:

ϕ=1 · 14.6%
ϕ=2 · 14.0%
ϕ=3 · 12.7%

Gate count 358 · 155 SX · 130 RZ · 69 CZ · 4 X

Three rules for NISQ circuits

01 **Ancilla-free by default.** Don’t add qubits that sit outside the main unitary.

02 **Every measured qubit earns it.** If you read it, it should take part in a controlled op.

03 **Own the transpiler.** Pin the layout. Don’t let it spread across the chip.

> And on the math side: a quantum spectral approach to **BSD** — encoding H_E, estimating eigenvalues, reading rank from zero-eigenvalue signatures — is at least **executable on real hardware today**.

Limits remain: depth near decoherence, 2-bit clock precision, hardware noise.

The constraint wasn't **the algorithm.** It was the orphan.

Remove the qubit the unitary never touched, hold the transpiler to the qubits you need — and HHL runs on NISQ.

> Have you hit transpiler or measurement edge cases on IBM hardware that the docs don't warn you about? I'd like to compare notes. ↓

HHL · 11a1 · IBM Torino · 5 qubits · 1024 shots

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